Let $\mathcal{P}$ denote the probability of selecting $(r_1, \ldots, r_n)$ as samples, or equivalently, $(r_1, \ldots, r_n)$ being rows with the top $n$ largest keys, where $r_1$ has the largest key, $r_2$ has the second largest key, etc. We wish to show $\mathcal{P} = \prod_{j=1}^{n} \left( \frac{w_j}{\sum_{k=j}^{N} w_k} \right)$.

Recall the key for the $j$-th row (denoted as $x_j$ from now on) is sampled from a probability distribution on $(-\infty, 0)$ with CDF $F_j(x) = e^{w_j x}$, and therefore the PDF of $x_j$ is $f_j(x) = F_j'(x) = w_j e^{w_j x}$. Given that $x_1 \geq x_2 \geq \cdots \geq x_n$, and also, $x_n \geq x_j$ for $j \in \{n+1, \ldots, N\}$, we have

$$\mathcal{P} = \int_{-\infty}^{0} f_1(x_1) \int_{-\infty}^{x_1} f_2(x_2) \cdots \int_{-\infty}^{x_{n-1}} f_n(x_n) \int_{-\infty}^{x_n} f_{n+1}(x_{n+1}) \cdots \int_{-\infty}^{x_n} f_N(x_N) dx_N \cdots dx_2 dx_1$$

Working through the multiple integrals above with ‘···’ in between and with intermediate steps containing more ‘···’s would be a bit too hand-wavy! So, in the interest of greater clarity, let’s be more pedantic and re-define $\mathcal{P}$ iteratively with $\mathcal{P}_0, \mathcal{P}_1, \ldots, \mathcal{P}_n$ instead. Let

$$\mathcal{P}_0(x_n) = \int_{-\infty}^{x_n} f_{n+1}(x_{n+1}) \cdots \int_{-\infty}^{x_n} f_N(x_N) dx_N \cdots dx_{n+1}$$

$$= \prod_{j=n+1}^{N} \left( \int_{-\infty}^{x_n} f_j(x_j) dx_j \right) = \prod_{j=n+1}^{N} \left( F_j(x) \bigg|_{-\infty}^{x_n} \right)$$

$$= \prod_{j=n+1}^{N} \left( e^{w_j x} \bigg|_{-\infty}^{x_n} \right) = e^{\left( \sum_{j=n+1}^{N} w_j \right) x_n}$$

(i.e., $\mathcal{P}_0(x_n)$ is the inner-most bunch of integrals where the integrands are functions of $x_{n+1}, \ldots, x_N$),

and then define

$$\mathcal{P}_j(x_{n-j}) = \int_{-\infty}^{x_{n-j}} f_{n-j+1}(x_{n-j+1}) \mathcal{P}_{j-1}(x_{n-j+1}) dx_{n-j+1}$$

for $j \in \{1, \ldots, n-1\}$, then it follows

$$\mathcal{P}_n(x_0) = \int_{-\infty}^{x_0} f_1(x_1) \mathcal{P}_{n-1}(x_1) dx_1$$

and $\mathcal{P} = \mathcal{P}_n(0)$.
Let’s then find out what \( P_1 \) is. By definition:

\[
P_1(x_{n-1}) = \int_{-\infty}^{x_{n-1}} f_n(x_n)P_0(x_n)dx_n = \int_{-\infty}^{x_{n-1}} w_n \cdot e^{w_n \cdot x_n} \left[ e^{\left( \sum_{j=n+1}^{N} w_j \right) x_n} \right] dx_n
\]

\[
= w_n \int_{-\infty}^{x_{n-1}} e^{\left( \sum_{j=n}^{N} w_j \right) x_n} dx_n = \left[ w_n / \left( \sum_{j=n}^{N} w_j \right) \right] \cdot e^{\left( \sum_{j=n}^{N} w_j \right) x_{n-1}}
\]

Now all that is remaining is simply an exercise of proof by mathematical induction, where given the induction hypothesis

\[
P_j(x_{n-j}) = \left[ \prod_{h=n-j+1}^{n} \left( w_h / \left( \sum_{k=h}^{N} w_k \right) \right) \right] \cdot e^{\left( \sum_{k=n-j+1}^{N} w_k \right) x_{n-j}}
\]

, which is true for \( j = 1 \), we shall show it is true for \( j \in \{2, \ldots, n\} \).

Suppose the inudction hypothesis is true for \( j = I - 1 \), then by definition

\[
P_I(x_{n-I}) = \int_{-\infty}^{x_{n-I}} f_{n-I+1}(x_{n-I+1})P_{I-1}(x_{n-I+1})dx_{n-I+1}
\]

\[
= \int_{-\infty}^{x_{n-I}} w_{n-I+1} \cdot e^{w_{n-I+1} \cdot x_{n-I+1}} \cdot \left[ \prod_{h=n-I+2}^{n} \left( w_h / \left( \sum_{k=h}^{N} w_k \right) \right) \right] \cdot e^{\left( \sum_{k=n-I+1}^{N} w_k \right) x_{n-I+1}} dx_{n-I+1}
\]

\[
= w_{n-I+1} \cdot \left[ \prod_{h=n-I+2}^{n} \left( w_h / \left( \sum_{k=h}^{N} w_k \right) \right) \right] \cdot \int_{-\infty}^{x_{n-I}} e^{\left( \sum_{k=n-I+1}^{N} w_k \right) x_{n-I+1}} dx_{n-I+1}
\]

\[
= \left[ w_{n-I+1} / \left( \sum_{k=n-I+1}^{N} w_k \right) \right] \cdot \left[ \prod_{h=n-I+2}^{n} \left( w_h / \left( \sum_{k=h}^{N} w_k \right) \right) \right] \cdot \left[ e^{\left( \sum_{k=n-I+1}^{N} w_k \right) x_{n-I+1}} \right]_{-\infty}^{x_{n-I+1}}
\]

\[
= \left[ \prod_{h=n-I+1}^{n} \left( w_h / \left( \sum_{k=h}^{N} w_k \right) \right) \right] \cdot e^{\left( \sum_{k=n-I+1}^{N} w_k \right) x_{n-I}}
\]

which shows the induction hypothesis is true for \( j = I \).
Therefore

\[ P_j(x_{n-j}) = \left[ \prod_{h=n-j+1}^{n} \left( \frac{w_h}{\sum_{k=h}^{N} w_k} \right) \right] \cdot e^{\left( \sum_{k=n-j+1}^{N} w_k \right) x_{n-j}} \]

for \( j \in \{1, \ldots, n\} \) and

\[ P = P_n(0) = \left[ \prod_{h=1}^{n} \left( \frac{w_h}{\sum_{k=h}^{N} w_k} \right) \right] \cdot e^{\left( \sum_{k=1}^{N} w_k \right) 0} = \prod_{h=1}^{n} \left( \frac{w_h}{\sum_{k=h}^{N} w_k} \right) \]

QED.